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corrosion at the tips of microcracks in the specimen. This causes the cracks to lengthen. After a period of time under a sustained tension, a crack reaches a critical length [Jaeger, 1962, p. \$5] and propagates unstably, causing fracture of the specimen.

Charles [1958] considered a highly elliptical hole of major axis L in a flat glass plate subject to an average tensile-stress S_y in a direction perpendicular to the major axis of the crack. He suggested

$$\dot{L}_x = f(S_x) + k \tag{19}$$

where \dot{L}_x is the velocity of the crack in the x direction. S_x is the tensile stress at the tip of the crack tangential to the crack surface; k is the corrosion rate at zero tangential stress. Suppose

$$f(S_r) = c(S_r/S_{cr})^n \tag{20}$$

where n is a positive constant, c is the maximum velocity of the crack, and S_{er} is the tensile strength of the atomic bonds at the crack tip. As

$$S_x/S_y = 2(L/r)^{1/2}$$
 (21)

$$S_{cr}/S_{u} = 2(L_{cr}/r)^{1/2}$$
 (22)

where r is the curvature at the crack tip, L_{er} is the length at which occurs the critical stress S_{er} for rupture of bonds at the crack tip. Equations 21 and 22 are derived from the theory of stress concentrations around holes in perfectly elastic bodies [Jaeger, 1962, p. 85].

Substituting equations 20 and 21 in equation 19 gives equation 23,

$$\dot{L}_x = c (L/L_{cr})^{n/2} + k \tag{23}$$

Charles suggested that, if static fatigue is to take place, stress-activated corrosion must occur at a much greater rate than stress-free corrosion. Hence the crack would grow with constant curvature r until it reaches its critical length L_{cr} . If stress-free corrosion were as important as stress-activated corrosion, the crack growth would occur with increasing radius of curvature, and the stress concentration might be seriously reduced.

The corrosion rate at zero stress k can therefore be neglected in equation 23 by comparison with the stress-dependent corrosion rate.

The temperature dependence of crack growth can be introduced by the assumption that corrosion is a rate process with an activation energy A. The experimentally determined activation energy of the process below 150° C is close to that for the diffusion of sodium atoms in glass. Charles suggested that the sodium atoms catalyse the hydrolysis of the oxygen-silicon bond in glass by creating free hydroxyl ions. Equation 23 can be written

$$\dot{L}_x = B(L/L_{cr})^{n/2} \exp(-A/KT)$$
 (24)

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Integrating equation 24 with respect to time gave equation 25,

$$\int_{L_0}^{L_{er}} dL (L/L_{er})^{-n/2} = \int_0^{t_f} B \exp(-A/KT) dt$$
$$[2L_{er}/(n-2)][(L_{er}/L_0)^{(n-2)/2} - 1]$$
$$= B \exp(-A/KT) t_f$$
(25)

When n and t_t are large, equation 25 can be written,

$$[2L_{cr}/B(n-2)]$$

$$\cdot \exp(A/KT)(L_{cr}/L_0)^{(n-2)/2} = t_f$$
(26)

Taking logarithms of equation 26,

$$\log t_f = (n/2) \log L_{er} - \log D$$
 (27)

where

$$D = L_0^{(n-2)/2} (B/2)(n-2) \exp (A/KT)$$

Equation 22 can be rewritten as equation 28,

$$L_{cr} = r S_{cr}^{2} / 4 S_{y}^{2} \tag{28}$$

Substituting equation 28 in equation 27,

$$\log t_f = -n \log S_y - \log D' \tag{29}$$

where

$$D' = (r S_{cr}^{2}/4)^{-n/2} \cdot D$$

Equation 29 gave the static-fatigue law (equation 18). The parameter n can be determined from the slope of a log $t_t - \log S_r$ plot. Charles [1958] reported a value of about 16.

The growth of subcritical cracks under tension has now been directly observed in glass microscope slides [*Wiederhorn*, 1967] and in sapphire [*Wiederhorn*, 1968].

It was found that the growth of a crack can be divided into two stages: a stage where crack